

B.TECH. I Year(R09) Regular Examinations, May/June 2010

MATHEMATICAL METHODS

(Common to Computer Science & Engineering, Electronics & Communication Engineering, Electrical & Electronics Engineering, Electronics & Instrumentation Engineering, Electronics & Computer Engineering, Electronics & Control Engineering, Information Technology, Computer Science & Systems Engineering)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions
All questions carry equal marks

1. (a) Reduce the matrix $A = \begin{pmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{pmatrix}$ to an Echlon form and hence

find its rank.

- (b) Find two non-singular matrices P and Q such that PAQ will be in the normal form

where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$

2. (a) Prove that the eigen values of an orthogonal matrix are of unit modulus.
(b) Reduce the following quadratic form to canonical form by Lagrange's reduction.
 $x_1^2 + 5x_2^2 + 9x_3^2 - 2x_1x_2 + 10x_2x_3 + 2x_1x_3$ and hence find the index, signature and nature of the quadratic form.
3. (a) Find the root of the equation $x^3 - 5x + 1 = 0$ using the Bisection method in 5 stages.
(b) By using Regula-Falsi method, find an approximate root of the equation $x^4 - x - 10 = 0$ that lies between 1.8 and 2. Carry out three approximations.
4. (a) Find by Taylor's series method the value of y at $x=0.1$ to five places of decimals from $\frac{dy}{dx} = x^2y - 1, y(0) = 1$.
(b) Find the value of y at $x=0.1$ by Picard's method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$.
5. (a) Define a Fourier series and write the Dirichlet conditions for the expansion of f(x) as a Fourier Series $(\alpha, \alpha + 2\pi)$.
(b) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ as a Fourier integral.
- Hence evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$.
6. (a) Form the partial differential equation by eliminating the arbitrary constants a, b and c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
(b) Solve the PDE by the method of separation of variables
 $y^3 z_x + x^2 z_y = 0$.
7. (a) Prove that Z-transform is linear.
(b) Find (i) $Z(na^n)$.
(ii) $Z(n^2 a^n)$.
8. (a) Fit a straight line to the following data

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.5

by the method of least squares.

- (b) For the following data, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0256

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- Apply elementary transformations to find the rank of $A = \begin{pmatrix} 1 & -7 & 3 & -3 \\ 7 & 20 & -2 & 25 \\ 5 & -2 & 4 & 7 \end{pmatrix}$
 - Compute the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 3 & -2 \\ 2 & 0 & -4 \end{pmatrix}$ by Gauss-Jordan method.
- Prove that the eigen values of a Hermitian matrix are all real.
 - Reduce the following quadratic form to canonical form by Lagrange's reduction $x^2 - 14y^2 + 2z^2 + 4xy + 16yz + 2zx$ and hence find the index, signature and nature of the quadratic form.
- Find and approximate value of the real root of $x^3 - x - 1 = 0$ using the Bisection Method.
 - Find the root of the equation $x \log_{10}(x) = 1.2$, using false position method.
- Using modified Euler's method, find an approximate value of y when $x=0.3$, given that $\frac{dy}{dx} = x + y$, $y(0) = 1$.
- Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$ in $0 < x < 2\pi$.
 - Express $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$ as a Fourier sine integral.
- Form the partial differential equation by eliminating the arbitrary constants a and b if $4(1 + a^2)z = (x + ay + b)^2$.
 - Solve by the method of separation of variables $4u_x + u_y = 3u$ and $u(0, y) = e^{-5y}$.
- State and prove damping rule for Z-transform.
 - Find $z^{-1} \left\{ \frac{z}{z^2 + 11z + 24} \right\}$.
- Fit the straight line to the following data

x	0.0	0.2	0.4	0.6	0.8	1.3
y	-1.85	-1.20	-0.55	0.15	0.80	1.35

by the method of least squares.

- A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time t seconds. Find the velocity of the slider and its acceleration when $t = 0.3$ seconds.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

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1. (a) Find the rank of the matrix $A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$

(b) If $A = \begin{pmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, find A^{-1} .

2. (a) Prove that the eigen values of a real symmetric matrix are all real.
(b) Reduce the following quadratic form to canonical form by Lagrange's reduction.
 $xy + y^2 + 4xz + z^2$ and hence find the index, signature and nature of the quadratic form.
3. (a) Find out the square root of 25 given $x_0 = 2.0$, $x_1 = 7.0$ using bisection method.
(b) Find out the roots of the equation $x^3 - x - 4 = 0$ using false position method.
4. (a) Obtain Picard's second approximate solution of the initial value problem
 $\frac{dy}{dx} = \frac{x^2}{y^2+1}$, $y(0) = 0$.
(b) Using the Taylor's series method, solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1$.
5. (a) Obtain the Fourier series to represent $f(x) = e^{ax}$ in $0 < x < 2\pi$.
(b) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that
 $F\{f(x) \cos ax\} = \frac{1}{2}\{F(s+a) + F(s-a)\}$.
6. (a) Obtain the partial differential equation by eliminating the arbitrary functions from $z = yf(x) + xg(y)$.
(b) Solve by the method of separation of variables $\frac{du}{dx} = 2\frac{du}{dt} + u$, given $u(x, 0) = 6e^{-3x}$.
7. (a) Find $z \left\{ \frac{1}{(n+1)(n+2)} \right\}$.
(b) Solve the difference equation using Z-transform $u_{n+2} - 3u_{n+1} + 2u_n = 0$, given that $u_0 = 0, u_1 = 1$.
8. (a) If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = mW + c$ connecting P and W using the following data:

P	12	15	21	25
W	50	70	100	120

Where P and W are taken in kg-wt. Compute P when $W = 150$ Kg.

- (b) For the following data, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

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- (a) Find the constants 'l' and 'm' such that the rank of the matrix $\begin{pmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & l & m \end{pmatrix}$ is (i) 3 (ii) 2

(b) Find A^{-1} when $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & -1 & 1 \\ -3 & 2 & 1 \end{pmatrix}$ by Gauss-Jordan method.
- (a) Prove that the eigen values of a skew Hermitian matrix are either purely imaginary or zero.

(b) Reduce the following quadratic form to canonical form by the diagonalisation method. Write also the corresponding linear transformation. Find the index, signature and nature of the quadratic form
 $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$.
- (a) By using bisection method, find an approximate root of the equation $\sin x = \frac{1}{x}$ that lies between $x=1$ and $x=1.5$ (measured in radians). Carry out computation upto 7th stage.

(b) Find the root of the equation $2x - \log_{10}x = 7$, which lies between 3.5 and 4 by Regula - falsi method. (or) Find the real root of the equation $2x - \log x = 7$, by successive approximate method.
- Solve by the Taylor's series method of third order problem
 $\frac{dy}{dx} = (x^3 + xy^2)e^{-x}$, $y(0) = 1$ for $x = 0.1, 0.2, 0.3$.
- (a) Find a Fourier series to represent $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$.
Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$.

(b) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that $F\{f(x - a)\} = e^{isa}F(s)$.
- (a) Form the PDE by eliminating the arbitrary function ϕ from the relation
 $\phi(x^2 + y^2 + z^2, lx + my + nz) = 0$.

(b) Solve by the method of separation of variables
 $u_x = 4u_y$, $u(0, y) = 8e^{-3y}$.
- (a) Find $z \{(\cos \theta + i \sin \theta)^n\}$. Hence evaluate $Z(\cos n\theta)$ and $Z(\sin n\theta)$.

(b) Find $z^{-1} \left\{ \frac{3z^2 + z}{(5z-1)(5z+2)} \right\}$.
- (a) Fit a straight line to the following data.

x	4	6	8	10	12
y	13.72	12.90	12.01	11.14	10.31

- (b) Evaluate approximately, by trapezoidal rule, $\int_0^1 (4x - 3x^2) dx$.

By taking $n = 10$. Compute the exact integral and find the absolute and relative error.
